# Summer Workshop on Arithmetic Geometry

July 22–25, 2013

Mathematical Sciences Center, Tsinghua University
Morningside Center of Mathematics, Chinese Academy of Sciences

### Venue

**Lecture Hall, Third Floor, Jinchuanyuan West Building, Tsinghua University**

## Schedule

### Monday July 22

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| 9:30–10:30 | **Qile Chen** (Columbia University)<br>Decomposition formula for log stable maps |
| 11:00–12:00 | **Yong Hu** (Universität Duisburg-Essen)<br>Arithmetic of quadratic forms over 2-dimensional local rings |
| 14:00–15:00 | **Jie Xia** (Columbia University)<br>Definition of Shimura curves in positive characteristic |
| 15:30–16:30 | **Yuhao Huang** (University of California, Berkeley)<br>Stability conditions and wall-crossing on K3 surfaces |

### Tuesday July 23

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| 9:30–10:30 | **Peng Shan** (CNRS & Université de Caen)<br>Representations of rational Cherednik algebras and affine Lie algebras |
| 11:00–12:00 | **Shanwen Wang** (Università degli Studi di Padova)<br>$p$-adic Eisenstein class and $p$-adic Eisenstein series |
| 14:00–15:00 | **Haoran Wang** (Université Paris 6)<br>The tame cover of the Drinfeld upper-half plane |
| 15:30–16:30 | **Jilong Tong** (Université de Bordeaux 1)<br>Néron model of curves |

## Wednesday July 24

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| 9:30–10:30 | **Yongqi Liang** (Université Paris 7)<br>Zero-cycles on varieties fibred over curves by Châtelet surfaces |
| 11:00–12:00 | **Han Wu** (École Polytechnique Fédérale de Lausanne)<br>Spectral decomposition and harmonic analysis: not a tautology |
| 14:00–15:00 | **Lei Zhang** (Boston College)<br>Eisenstein series on covers of odd orthogonal groups |
| 15:30–17:00 | **Yannan Qiu** (National University of Singapore)<br>Siegel-Weil formula and its regularization<br>(Lecture 1 of 5, not part of the workshop) |

## Thursday July 25

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| 9:30–10:30 | **Zhi Jiang** (Université Paris-Sud)<br>Pseudoeffective classes and pushforwards |
| 11:00–12:00 | **Xu Shen** (Université Paris-Sud)<br>On the $p$-adic geometry of Rapoport-Zink spaces |

# Abstracts

**Qile Chen**
**Decomposition formula for log stable maps**

Consider the Gromov-Witten invariants under a one parameter log smooth degeneration. In a joint project with Abramovich-Gross-Siebert, we show that the Gromov-Witten invariants of the fibers can be decomposed to a sum of log stable maps with specified combinatorial type given by marked graphs. This is the first step of our project on study degeneration formulas in general.

**Yong Hu**
**Arithmetic of quadratic forms over 2-dimensional local rings**

Let $R$ be a 2-dimensional henselian local domain, and let $K$ be its fraction field. I will talk about two arithmetic problems about quadratic forms over $K$: (1) what is the maximal dimension of quadratic forms with no (nontrivial) zeros over $K$? (2) does the local-global principle hold for forms of low dimensions? If time allows, I'll also discuss some applications to the arithmetic of semisimple groups.

**Yuhao Huang**
**Stability conditions and wall-crossing on K3 surfaces**

For triangulated categories, Bridgeland defined the notion of stability conditions and showed the set of certain stability conditions form a complex manifold. The automorphism group of the derived category of K3 surfaces is determined by the geometry of one connected component of the stability manifold, which is unknown. Fix a finite set of objects, one has a wall and chamber structure on the stability manifold so that stable objects are destabilized only when one cross a wall.

**Zhi Jiang**
**Pseudoeffective classes and pushforwards**

Given a morphism between complex projective varieties, we make several conjectures on the relations between the set of pseudo-effective (co)homology classes which are annihilated by pushforward and the set of classes of varieties contracted by the morphism. We prove these conjectures for classes of curves or divisors. We also prove that one of these conjectures implies Grothendieck's generalized Hodge conjecture for varieties with Hodge coniveau at least 1. This is a joint work with Olivier Debarre and Claire Voisin.

**Yongqi Liang**
**Zero-cycles on varieties fibred over curves by Châtelet surfaces**

Let $k$ be a number field. Consider the family of smooth projective $k$-varieties fibred over a curve by Châtelet surfaces. Poonen discovered that, for certain varieties in this family, it is not sufficient to explain the failure of Hasse principle and weak approximation for rational points by the Brauer-Manin obstruction. However, for this family of varieties, we prove that the Brauer-Manin obstruction is the only obstruction to the local-global principle for zero-cycles.

**Peng Shan**
**Representations of rational Cherednik algebras and affine Lie algebras**

To any complex reflection group $W$, one can associate a family of associative algebras called the rational Cherednik algebras. Their representations share some similarities with representations of semisimple Lie algebras. In particular they have a category $\mathbf{O}$ which is a highest weight category and is closely related to Hecke algebras associated with $W$. On the other hand, affine Lie algebras are natural generalizations of semisimple Lie algebras. When $W$ is the wreath product of a symmetric group and a cyclic group, the similarity mentioned above can be made precise – there is actually an equivalence of categories between the category $\mathbf{O}$ of rational Cherednik algebras and the parabolic category $\mathbf{O}$ of affine Lie algebras of $\mathfrak{gl}_N$. In this talk, we will explain this equivalence, and some important consequences of it such as the character formulae for simple modules of these Cherednik algebras and the Koszulity of its category $\mathbf{O}$. This is a joint work with R. Rouquier, M. Varagnolo and E. Vasserot.

**Xu Shen**
**On the $p$-adic geometry of Rapoport-Zink spaces**

We will talk about $p$-adic geometry of Rapoport-Zink spaces. These spaces are moduli of $p$-divisible groups, and they are local analogues of PEL type Shimura varieties. For the Rapoport-Zink spaces which satisfy the Hodge-Newton condition, we prove that these spaces can be obtained as some kind of parabolic inductions, in particular their cohomology contain no supercuspidal representations. For the supersingular Rapoport-Zink spaces, we try to find some fundamental domains for the related group actions on them. We will concentrate on some unitary group cases, where one proves such fundamental domains indeed exist, by using Fargues's theory of Harder-Narasimhan filtration for finite flat group schemes. We will talk also some applications to cohomology of these geometric constructions.

**Jilong Tong**
**Néron model of curves**

Let $R$ be a Dedekind domain, and $K$ its fraction field. Let $X_K$ be a smooth $K$-variety. Its Néron model is a smooth $R$-model of $X_K$ verifying the so-called Néron universal property. When $X_K$ is an abelian variety, it is well-known that its Néron model always exists. In this talk, we will discuss the existence of Néron model of a smooth curve. This talk is based on a joint work still in progress with Liu Qing in Bordeaux.

**Haoran Wang**
**The tame cover of the Drinfeld upper-half plane**

In this talk, we will describe the geometry and cohomology of the tamer cover of the Drinfeld upper-half space and its relation with Deligne-Lusztig varieties. In particular, this will give a realization of the local Langlands correspondence and the Jacquet-Langlands correspondence for depth 0 supercuspidal representations in a purely local way. For the non supercuspidal part, we can determine the correspondences up to a contragredient.

**Shanwen Wang**
**$p$-adic Eisenstein class and $p$-adic Eisenstein series**

G. Kings studied the étale realisation of the polylogarithm and gave an explicit description of this realisation. Recently, René Scheider gave a de Rham realisation of the polylogarithm. We construct an explicit reciprocity law, which gives a relation between these two realisations.

**Han Wu**
**Spectral decomposition and harmonic analysis: not a tautology**

We present a basic problem in the representation theory: Spectral Decomposition. We first review it in the very classical setting, i.e. the Fourier analysis. We then generalize it by proposing a framework. We stress on the analogous part, that we name as Harmonic Analysis, which interests analysts more than representation theorists. Finally, we recollect some results in the case of automorphic representation theory for $\mathbb{GL}_2$ and discuss its general form for any reductive algebraic group, a statement "well-known to experts" but actually a conjecture. This last part served as a basic tool in my thesis and was developed there.

**Jie Xia**
**Definition of Shimura curves in positive characteristic**

Classical Shimura varieties are defined over complex field and generally have number fields as the field of definition. In this talk, we find conditions which guarantee a curve in char. $p$ lifts to a Shimura curve of Hodge type. The conditions, in terms of $p$-divisible groups or crystalline Hodge classes, are intrinsic in positive characteristic. So we can take them as the definition of Shimura curves in char. $p$. The key ingredient will be a deformation result on versally deformed Barsotti-Tate groups.

**Lei Zhang**
**Eisenstein series on covers of odd orthogonal groups**

We study the Whittaker coefficients of the minimal parabolic Eisenstein series on the $n$-fold cover of the split odd orthogonal group $\mathrm{SO}(2r+1)$. If the degree of the cover is odd, then Beineke, Brubaker and Frechette have conjectured that the $p$-power contributions to the Whittaker coefficients may be computed using the theory of crystal graphs of type C, by attaching to each path component a Gauss sum or a degenerate Gauss sum depending on the fine structure of the path. We establish their conjecture using a combination of automorphic and combinatorial-representation-theoretic methods. Surprisingly, we must make use of the type A theory, and the two different crystal graph descriptions of Brubaker, Bump and Friedberg available for type A based on different factorizations of the long word into simple reflections. We also establish a formula for the Whittaker coefficients in the even degree cover case, again based on crystal graphs of type C. As a further consequence, we establish a Lie-theoretic description of the coefficients for $n$ sufficiently large, thereby confirming a conjecture of Brubaker, Bump and Friedberg.