

Note on “Parity and symmetry in intersection and ordinary cohomology” [SZ16]

As mentioned before [SZ16, Proposition 3.3.12], the results of [SZ16, Section 3.2] have analogues over an arbitrary field k in which ℓ is invertible.

Definition. Let X be a Deligne-Mumford stack of finite presentation over k . We say that $A \in D_c^b(X, \overline{\mathbb{Q}}_\ell)$ is *split admissible* if it is split [SZ16, Definition 3.2.1] and if the pullback of A to $X \otimes_k \bar{k}$ is admissible semisimple [SZ16, Definition 3.3.1].

Let $\sigma = \pm 1$, $w \in \mathbb{Z}$. We denote by $D_\sigma^w(X, \overline{\mathbb{Q}}_\ell) \subseteq \text{Ob}(D_c^b(X, \overline{\mathbb{Q}}_\ell))$ the subset consisting of split admissible complexes A such that ${}^p\mathcal{H}^i A$ is $(-1)^{w+i}\sigma$ -self-dual with respect to $K_X(-w-i)$ for all i . We denote by $D_d^w(X, \overline{\mathbb{Q}}_\ell) \subseteq \text{Ob}(D_c^b(X, \overline{\mathbb{Q}}_\ell) \times D_c^b(X, \overline{\mathbb{Q}}_\ell))$ the subset consisting of pairs (A, B) of split admissible complexes such that ${}^p\mathcal{H}^i A$ is isomorphic to $(D_X {}^p\mathcal{H}^i B)(-w-i)$ for all i .

The following is a generalization of [SZ16, Theorems 3.2.3, 3.3.7]. The proof is the same as in [SZ16], with the relative hard Lefschetz theorem provided by [SZ16, Proposition 3.3.10].

Theorem. *Let $f: X \rightarrow Y$ be a proper morphism of Deligne-Mumford stacks of finite presentation over k , where Y has finite inertia. Then Rf_* preserves D_σ^w and D_d^w .*

In particular, Rf_* preserves split admissible complexes.

Corrections to [SZ16]. Definition A.2.1: Read “equal” for “isomorphic”.

In the proof of Proposition 4.3.1, instead of [Del80, 1.7.9] which depends on global equations, one should use the more canonical construction of [Del80, 1.7.10]. The assumption on the existence of global equations is thus superfluous.

Reference

- [Del80] P. Deligne, *La conjecture de Weil. II*, Inst. Hautes Études Sci. Publ. Math. **52** (1980), 137–252 (French). MR601520 (83c:14017)
- [SZ16] S. Sun and W. Zheng, *Parity and symmetry in intersection and ordinary cohomology*, Algebra Number Theory **10** (2016), no. 2, 235–307, DOI 10.2140/ant.2016.10.235.